Workshop on Functional Analysis and its Applications

(January 14-16, 2022)

Titles and Abstracts

Department of Applied Sciences
IIIT Allahabad

Perturbation analysis of discrete eigenvalues of bounded linear operators

Rafikul Alam
IIT Guwahati

Abstract. Let X be a complex Banach space and let $T : X \longrightarrow X$ be a bounded linear operator with nonempty discrete spectrum $\sigma_d(T)$. For a bounded linear operator $V : X \longrightarrow X$, we consider a one parameter family of bounded linear operators given by $T(t) := T + tV$ for $t \in \mathbb{C}$. We discuss evolution of discrete eigenvalues of $T(t)$ when t varies in $\mathbb{C}$.

Application of positive linear operators in finding the Hausdorff Dimension

Dr. Amit Priyadarshi,
IIT Delhi

Abstract. In this talk we will talk about attractor of an iterated function system (IFS) and its Hausdorff dimension. We will see how the concept of spectral radius of positive linear operators will play a role in obtaining a formula for the Hausdorff dimension of the attractor. As an illustration, we will use the formula to obtain lower bounds for the Hausdorff dimension of a set of complex continued fractions.

Various Smoothness Properties in Banach Spaces

Vamsinadh Thota
NIT Tiruchirappalli

Abstract. In this talk, we will discuss various differentiability notions of the norm function. We also explore some equivalent characterizations of these differentiablity notions in terms of rotundity properties.

Orthogonality in some Banach spaces

Priyanka Grover,
Shiv Nadar University

Abstract. A well studied notion of orthogonality in normed spaces is that of Birkhoff James orthogonality. An element x is said to be (Birkhoff-James) orthogonal to a subspace W if $||x + w|| \geq ||x||$ for all $w \in W$. In a Hilbert space with inner product $\langle \cdot, \cdot \rangle$, if W is a one-dimensional subspace spanned by an element y, then this becomes the usual orthogonality $\langle x, y \rangle = 0$. The characterization of orthogonality has been extensively used lately to study norm parallelism, smooth points, distance formulas etc. We shall describe this concept in detail and see some of these conditions for M_n, the space of $n \times n$ matrices; $B(H)$, the space of bounded linear operators on a Hilbert space H; and $C(X)$, the space of continuous functions on a compact Hausdorff space X.

Continuity of linear operators commuting with convolution operators

Vishvesh Kumar
Ghent University, Belgium

Abstract. In 1967, B. E. Johnson conjectured the following problem:

"Let E_1, E_2 be two Banach spaces and let T_1, T_2 be two continuous linear operators on E_1, E_2 respectively. Suppose that S is a linear operator from E_1 into E_2 such that $ST_1 = T_2S$. If (T_1, T_2) has no critical eigenvalues then S is continuous."

In this expository talk, we will discuss some recent developments related with this conjecture. In particular, we will focus in the case when $E_2 = L^1(G)$ and T_2 is convolution operator on $L^1(G)$, i.e., $T_2 f := \mu * f$, where μ is a bounded Borel measure on a locally compact group G.

Some problems in discrete scattering and lattice mechanics

Basant Lal Sharma
IIT Kanpur

Abstract. The talk will feature the discrete analogue of Sommerfeld half-plane diffraction in three types of lattices (square, triangular, hexagonal) by a (finite or semi-infinite) line defect (a rigid constraint or a crack). Exact solutions are provided, in integral form, for the semi-infinite case of both types of defects. The talk will discuss square lattice mostly. The relation of finite defect to its semi-infinite counterpart, along with the existence and uniqueness of the latter has been proven on ℓ_2 spaces. The continuum limit of semi-infinite defect coincides with the classical Sommerfeld solution via convergence in certain Sobolev

spaces. Applications of these problems are wave scattering by sharp edged defects in meta-materials, high-frequency wave scattering in elastic media, fracture mechanics, electronic flow in graphene like materials, numerical solution of scattering of plane polarized E/M waves by conducting screens/edges or acoustic wave scattering by soft or hard screens/edges, etc. If time permits I will discuss some open problems in discrete scattering as well as other nonlinear problems in mechanics.

Projections in the convex hull of isometries on Banach spaces

Abdullah Bin Abu Baker
IIIT Allahabad

Abstract. A projection P on a Banach space X is called a generalized bi-circular projection if there exists a unit modulus complex number λ different from 1 such that $P+\lambda(I-P)$ is an isometry on X. In this talk, we shall discuss some properties of generalized bi-circular projections and their relationship with Hermitian projections. We will also look for projections that can be written as convex combination of isometries on some Banach spaces.

Fixed Point Theory on CAT(0) Spaces and an Application

Javid Ali
Aligarh Muslim University

Abstract. In this talk, first we recall some well-known fixed point results from the literature. Then we approximate fixed points of multi-valued non-expansive mappings in CAT(0) spaces. Finally, we utilize our main result to study image recovery problem in CAT(0) spaces.

A Geometric View of Hahn-Banach Extension Theorem

G Sankara Raju Kosuru
IIT Roper

Abstract. In this talk, we first state the Hahn-Banach Extension theorem. Further, we discuss some geometrical implications of the theorem.

Almost Invariant Subspaces of the Shift Operator on Vector-Valued Hardy Spaces

Soma Das
IIT Guwahati

Abstract. In this talk, I will briefly discuss about basic Hardy space and some useful results in Hardy space. As a nice application of these results, I will introduce our work in which we have characterized nearly invariant subspaces of finite defect for the backward shift operator acting on the vector valued Hardy space which is a vectorial generalization of a result of Chalendar–Gallardo–Partington. Using this characterization of nearly invariant subspace under the backward shift we completely describe the almost invariant subspaces for the shift and its adjoint acting on the vector valued Hardy space.

On Operators attaining norm on every reducing subspace

G Ramesh
IIT Hyderabad

Abstract. Let H be a complex Hilbert space and $\mathcal{B}(H)$ be the space of all bounded linear operators on H. We say $T \in \mathcal{B}(H)$ to be norm attaining if there exists $x \in H$ with $\|x\| = 1$ such that $\|Tx\| = \|T\|$. We define a new class

$$\beta(H) := \{T \in \mathcal{B}(H) : T \text{ attains norm on every reducing subspace of } T\}.$$

In this talk we discuss properties and structure of elements in $\beta(H)$.

This is a joint work with Prof. Hiroyuki Osaka, Ritsumeikan University, Japan.

Fixed Point Theory of nonexpasive Mappings and Proximal Point Algorithm

Izhar Uddin
Jamia Millia Islamia

Abstract. Due to its wide applications, fixed point theory of nonexpansive mappings is a very important topic in nonlinear analysis. In 1970, Martinet initiated the proximal point algorithm (shortly PPA) and after it Rockafellar developed PPA in Hilbert space and proved that this method converges to a solution of the convex minimization problem. Recently, it has been become a fascinating topic to extend PPA for solving optimization problem in the setting of manifolds which are extension of Hilbert, Banach and linear space. During this talk, we will discuss some basic theorems for nonexpanive mapping and some variants of PPA in different spaces.

Fractal operator corresponding to α-fractal functions

Sangita Jha
NIT Rourkela

Abstract. Barnsley introduced fractal interpolation functions (FIFs) to approximate rough functions in 1986. The FIFs with appropriate IFSs provide a method to perturb and approximate a continuous function on a compact interval I. This method produces a class of fractal functions f^α with respect to the scaling factor α. Further, the method of perturbation yields an operator known as a fractal operator. We discuss the basic properties of the associated fractal operator. As essential parameters of the IFS, the scaling factor α has important consequences in the properties of the function f^α. We also discuss the notion of the fractal dimension of α-fractal functions.

Norm Attainment of Toeplitz Operators

Kousik Dhara
Weizmann Institute of Science, Israel

Abstract. Suppose $\mathcal{H}$ is a Hilbert space. A bounded linear operator T on $\mathcal{H}$ is said to be norm attaining if there exists a unit vector $x_0 \in \mathcal{H}$ such that $\|Tx_0\| = \|T\|$. It is natural to study operators that attain the norm. It is known (and easy to prove) that the compact operators are always norm attaining. So one would like to look beyond compact operators. In this talk, we shall discuss about the norm attainment of Toeplitz operators on the vector valued Hardy space over the unit disc. This is based on a joint work with Neeru Bala, Aryaman Sensarma and Jaydeb Sarkar.

Trace formulae in Perturbation theory

Chandan Pradhan
IIT Guwahati

Abstract. Krein found a trace formula for perturbations of self-adjoint (also unitary) operators by trace class operators. Later Koplienko found a trace formula for perturbations of self-adjoint operators by Hilbert-Schmidt class operators. In 1988, a similar formula was obtained by Neidhardt in the case of unitary operators. In this presentation, we will briefly discuss all these trace formulas and present an alternative proof of the Koplienko-Neidhardt trace formula in the case of unitary operators by reducing the problem to a finite-dimensional one.